Design of machine element (2025502)

UNIT-1 Introduction to design

Design Procedure

1. Need or Aim or Objective
2. Mechanism or synthesis - Relative motion of component
3. Force analysis
4. Material Selection
5. Design of parts (Shape & size)
6. Testing and modifications.
7. Production
8. Assembly.

Load: Any external force or couple acting on a body or machine component or structural member is known as load. It is denoted as P or W.

Types of load

- Direct or Normal load:- If the load is acting perpendicular to the axis of plane, then it is known as direct load. Direct load causes to change the size of the body.
- Shear load:- If the the load is acting parallel to the axis of plane, then it is known as shear or transverse load. Shear load causes to change the shape of the body.

→ Tensile Load:- If the load is used to elongate the body, the it is known as tensile load.

→ Compressive Load:-
If the load is used to compress the body, then it is known as compressive load.

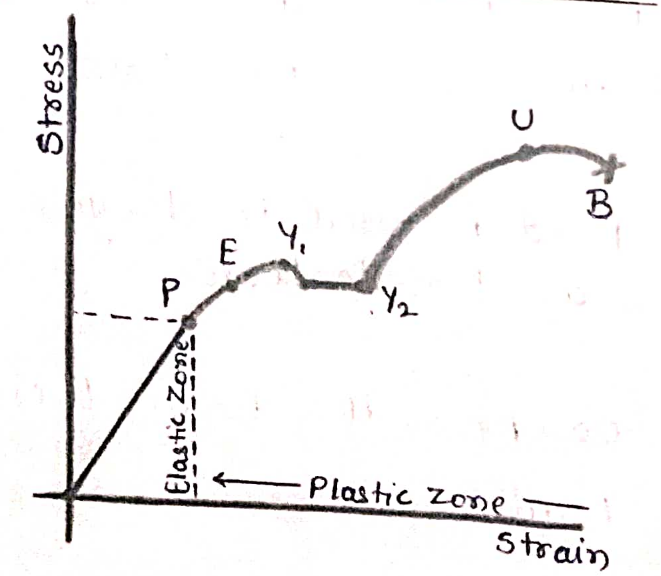
Mechanical Properties of material :-

1. Strength
2. Rigidity
3. Hardness
4. Toughness
5. Brittleness
6. Ductility
7. Elasticity
8. Plasticity.

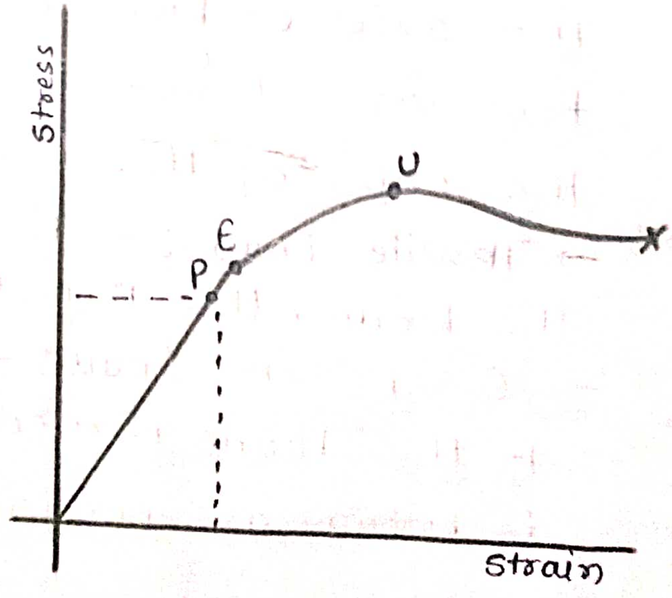
Ductile Material :- The material which is stretched or pulled out into a wire known as ductile material. ie. Large amount of plastic deformation take place between elastic limit or fracture point (breaking point).

Brittle material :- The material which can not be streched. In brittle material small amount of plastic deformation takes place between elastic limit and fracture point. Brittle material are cast iron, concrete, glass etc.

Stress - Strain Diagram



for ductile material.



for brittle material.

P denotes limit of Perptionality,

E denotes elastic limit,

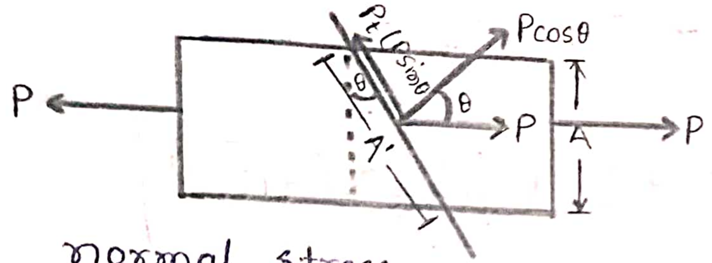
Y_1 denotes Upper yield point,

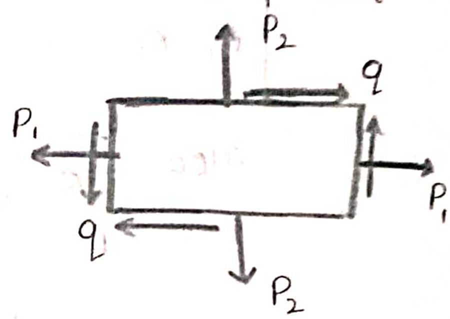
Y_2 denotes Lower yield point,

U denotes Ultimate point,

B denotes Breaking or fracture point.

Expression for stress on an oblique plane subjected to normal stress in two direction and shear stress:-





$A_1 = A/\cos\theta$

normal stress,

$$\sigma_N = \frac{P_N}{A_1} = \frac{P\cos\theta}{\frac{A}{\cos\theta}} = \frac{P}{A}\cos^2\theta = \sigma\cos^2\theta$$

Shear or tensile stress,

$$\sigma_t = \frac{P_t}{A_1} = \frac{-P\sin\theta}{\frac{A}{\cos\theta}} = -\frac{P}{A}\cdot\frac{2}{2}\cdot\sin\theta\cos\theta$$

$$= -\frac{\sigma}{2}\sin 2\theta$$

normal stress on oblique plane,

$$\boxed{\sigma_n = \left(\frac{P_1 + P_2}{2}\right) + \left(\frac{P_1 - P_2}{2}\right)\cos 2\theta + q\sin 2\theta}$$

also, $$\boxed{\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta}$$

Shear stress on oblique plane,

$$\boxed{\tau = -\left(\frac{P_1 - P_2}{2}\right)\sin 2\theta + q\cos 2\theta}$$

Also, $$\boxed{\tau = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta}$$

If $P_t = 0$, $P_n = max^m$ (principle plane).

$$\left(\frac{P_1 - P_2}{2}\right) \sin 2\theta = q \cos 2\theta$$

$$\boxed{\tan 2\theta = \left(\frac{2q}{P_1 - P_2}\right)}$$ also, $$\boxed{\tan 2\theta_p = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)}$$

$$\boxed{2\theta = \tan^{-1}\left(\frac{2q}{P_1 - P_2}\right)}$$

principle stress,

$$\boxed{\sigma_1, \sigma_2 = \left(\frac{P_1 + P_2}{2}\right) \pm \sqrt{\left(\frac{P_1 - P_2}{2}\right)^2 + q^2}}$$

Also, $$\boxed{\sigma_{max}\ \sigma_{min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}}$$

Maximum shear stress,

$$\boxed{\tau_{max} = \sqrt{\left(\frac{P_1 - P_2}{2}\right)^2 + q^2}}$$

Also, $$\boxed{\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}}$$

Qn. x - section = 50 mm X 50 mm
force, $P_1 = 4$ KN, $P_2 = 10$ KN, $q = 2$ KN, find σ_N & τ at 30° & 60°, θ_P ; σ_1, σ_2, τ_{max}.

$$\sigma_x = \frac{P_1}{A} = \frac{4 \times 10^3}{50 \times 50} = 1.6\ N/mm^2$$

$$\sigma_y = \frac{P_2}{A} = \frac{10 \times 10^3}{50 \times (50+50)} = 2\ N/mm^2$$

$$q = \frac{2 \times 10^3}{50 \times (50+50)} = 0.4\ N/mm^2$$

normal stress at $\theta = 30°$

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + q \sin 2\theta$$

$$= \left(\frac{1.6 + 2}{2}\right) + \left(\frac{1.6 - 2}{2}\right)\cos 60 + 0.4 \times \sin 60°$$

$$= 1.8 - (0.2)(0.5) + (0.4)(0.866)$$

$$= 1.8 - 0.1 + 0.3464$$

$$= 2.0464 \text{ N/mm}^2$$

Shear stress,

$$\tau = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + q \cos 2\theta$$

$$= -\left(\frac{1.6 - 2}{2}\right)\sin 60 + 0.4 \times \cos 60°$$

$$= (0.2)(0.866) + (0.4) \times (0.5)$$

$$= 0.1732 + 0.08$$

$$= 0.2532 \text{ N/mm}^2$$

Normal stress at $\theta = 30°$

$$\sigma_n = \left(\frac{1.6 + 2}{2}\right) + \left(\frac{1.6 - 2}{2}\right)\cos 120 + (0.4)\sin 120$$

$$= 1.8 + (-(0.1)) + 0.3464$$

$$= 2.2464 \text{ N/mm}^2$$

Shear stress,

$$\tau = -\left(\frac{1.6 - 2}{2}\right)\sin 120° + q \cos 120°$$

$$= 0.1732 + (-0.2)$$

$$= -0.0268 \text{ N/mm}^2$$

$$\tan 2\theta_p = \frac{2q}{\sigma_x - \sigma_y} = \frac{2 \times \cancel{0.4}}{-\cancel{0.4}} = -2$$

$$2\theta_p = \tan^{-1}(2)$$

$$2\theta_p = -63.43$$

$$\theta_p = -31.7174$$

1° ~~rad~~ =

$$= 180 + 31.71 = 211.71°$$

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + q^2}$$

$$= \frac{1.6 + 2}{2} + \sqrt{\left(\frac{1.6 - 2}{2}\right)^2 + (0.4)^2}$$

$$= 1.8 + \sqrt{0.4 + 0.16}$$

$$= 1.8 + 0.7483$$

$$= 2.5483 \ N/mm^2$$

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (0.4)^2}$$

$$= 1.8 - 0.7483$$

$$= 1.05167 \ N/mm^2$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + q^2}$$

$$= 0.7483 \ N/mm^2$$

P - forces

σ_x, σ_y - Stresses

q - Shear force

τ - Shear stress

σ_1, σ_2 - principle stress ($\sigma_{max}, \sigma_{min}$)

properties of material

S_{ut} - Ultimate tensile strength

S_{yt} - Yield tensile strength

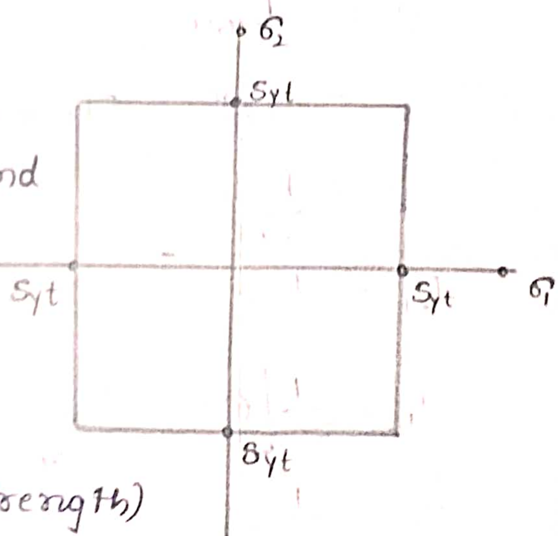
S_{sy} - Yield shear strength

e_x, e_y, e_z - Strain [unidirectional force, unidirectional load]

ϵ_1, ϵ_2 - Principle strain (produced dere to principal stress)

Theory of Failure

Any material is said to be failed when "stress > permissible stress", and there is permanent deformation.



1. Maximum Principle stress Theory (Rankine Theory)

$\sigma_{max} \leq S_{yt}$ (Yield tensile strength)

or $\sigma_{max} \leq S_{ut}$ (Ultimate shear strength)

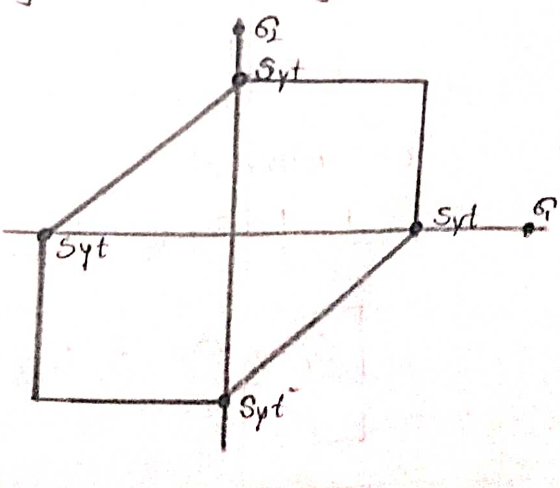
2. Maximum shear stress Theory (Tresa's theory)

$\tau_{max} \leq \tau_{permissible}$

$$\frac{\sigma_1 - \sigma_2}{2} \leq \frac{S_{yt}}{2}$$

$$\boxed{\sigma_1 - \sigma_2 \leq S_{yt}}$$

$\boxed{\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}}$, $\boxed{\tau_{per} = \frac{\sigma_x}{2} = \frac{S_{yt}}{2} = S_{sy}}$



3. Maximum Principle Strain Theory

$$\epsilon_{max} \le \epsilon_{permissible}$$

$e_1 \quad e_2 \quad e_3$

Biaxial stress; σ_1 & σ_2

$$E_1 = \frac{\sigma_1}{E} - \frac{\sigma_2}{mE} - \frac{\sigma_3}{mE} < \epsilon_{per} < \frac{S_{yt}}{2}$$

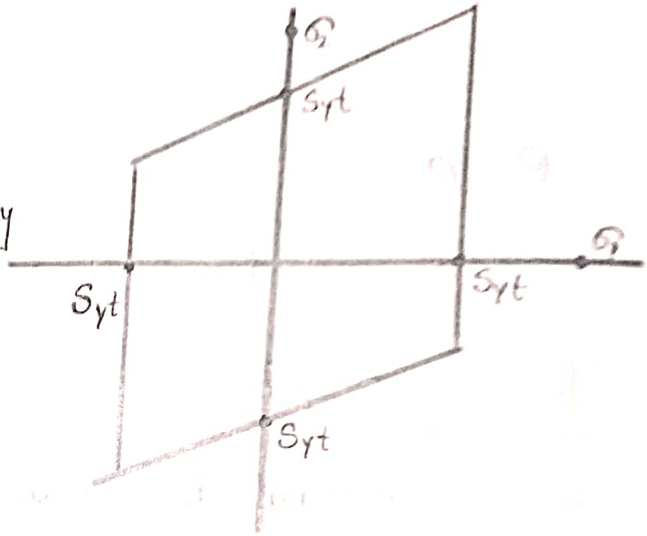
$$E_2 = \frac{\sigma_2}{E} - \frac{\sigma_1}{mE} - \frac{\sigma_3}{mE}$$

$$E_3 = \frac{\sigma_3}{E} - \frac{\sigma_1}{mE} - \frac{\sigma_2}{mE}$$

$$\frac{\sigma_1}{E} - \frac{\sigma_2}{mE} \le \frac{S_{yt}}{E}$$

$$\boxed{\sigma_1 - \frac{\sigma_2}{m} \le S_{yt}}$$

$$\epsilon_{max} = \frac{\sigma_1}{E} - \frac{\sigma_3}{mE}$$



4. Maximum strain Energy Theory (Haigh Theory);

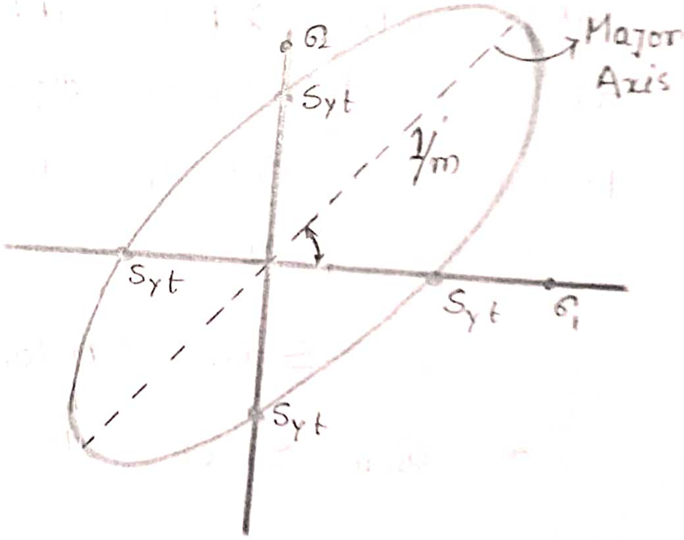
$$U = \frac{1}{2}\sigma\epsilon$$

$$= \frac{\sigma^2 V}{2E}$$

$$= \frac{P^2 L}{2AE}$$

Strain Energy

$$U_1 = \frac{1}{2}\sigma_1\epsilon$$



$$u_1 = \frac{1}{2E}\left[\sigma_1^2 - \frac{\sigma_1\sigma_2 + \sigma_1\sigma_3}{m}\right]$$

$$u_2 = \frac{1}{2E}\left[\sigma_2^2 - \frac{\sigma_2 \cdot \sigma_3 + \sigma_2\sigma_1}{m}\right]$$

$$u_3 = \frac{1}{2E}\left[\sigma_3^2 - \frac{\sigma_3\sigma_2 + \sigma_1\sigma_3}{m}\right]$$

$$u = u_1 + u_2 + u_3$$

$$u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \frac{2(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)}{m}\right] \le \frac{S_{yt}^2}{2E}$$

for biaxial,

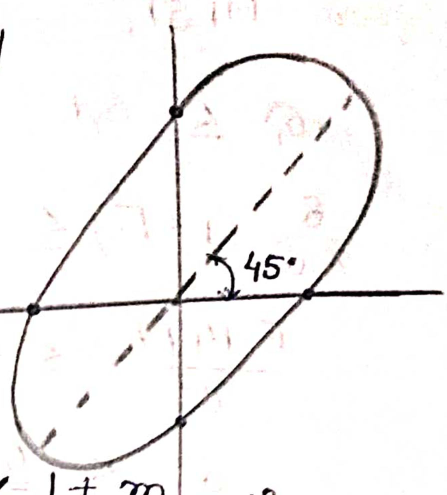
$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 - \frac{2\sigma_1\sigma_2}{m}\right] \le \frac{S_{yt}^2}{2E}$$

$$\boxed{\sigma_1^2 + \sigma_2^2 - \frac{2\sigma_1\sigma_2}{m} \le S_{yt}^2}$$

5. Maximum distortion energy theory

(Von-mises theory)

Distortion Energy = Strain Energy − Volumetric strain energy



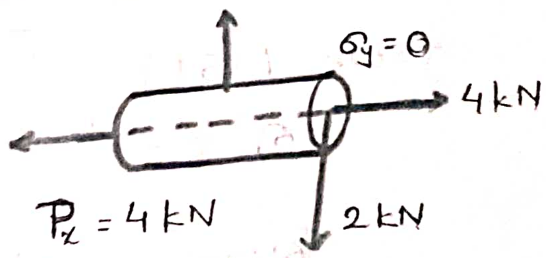
$$\frac{1+m}{6mE}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] \le \frac{1+m}{3mE} S_{yt}^2$$

for biaxial stress,

$$\frac{1}{2}\left[\sigma_1^2+\sigma_2^2-2\sigma_1\sigma_2+\sigma_2^2+\sigma_1^2\right] \le S_{yt}^2$$

$$\boxed{\sigma_1^2+\sigma_2^2-\sigma_1\cdot\sigma_2 \le S_{yt}^2}$$

Qn. A circular rod is being cooled by force of 4kN and it is also subjected to transverse shear load of magnitude 2kN. Find the diameter of rod using (I) MPST, (II) MSST, (III) MPST, (IV) MSET, (V) MDET. The yield tensile strength of Poison ratio, $\mu = 0.3$. The yield tensile strength of material is 450 N/mm².



$P_x = 4\text{kN}$

$P_y = 2\text{kN}$

$\sigma_x = \dfrac{4}{\frac{\pi d^2}{4}} = \dfrac{16}{\pi d^2}$

$q = \dfrac{8}{\pi d^2}$

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x+\sigma_y}{2}\right)^2+q^2}$$

$$= \frac{16}{\pi d^2\times 2} + \sqrt{\left(\frac{16}{\pi d^2\times 2}\right)^2+\left(\frac{8}{\pi d^2\times 2}\right)^2}$$

$$\sigma_1 = \frac{8}{\pi d^2}(1+\sqrt{2})$$

$$\sigma_2 = \frac{8}{\pi d^2}(1-\sqrt{2})$$

from MPST,

$$\sigma_1 \leq S_{yt}$$

$$\frac{8}{\pi d^2}(1+\sqrt{2}) \leq 450$$

$$\frac{6.1477}{d^2} \times 10^3 \leq 450$$

$$d^2 \leq \frac{6.1477}{450} \times 10^3$$

$$d^2 \leq 13.6615$$

$$d \geq \underline{3.6961}.$$

from MSST

$$\sigma_1 - \sigma_2 \leq S_{yt}$$

$$\frac{6.1477 + 1.05}{d^2} \leq 450$$

$$d^2 \leq \frac{6.1977}{450} \times 10^3$$

$$d^2 \leq 13.77267$$

$$d \geq \underline{3.711}$$

from MSET

$$\sigma_1^2 + \sigma_2^2 - \frac{2\sigma_1\sigma_2}{m} \leq S_{yt}^2$$

$$\left[\frac{6.147}{d^2}\right]^2 + \left[\frac{-1.05}{d^2}\right]^2 - 2 \cdot \frac{6.147 \cdot -1.05}{d^2 \cdot d^2} \cdot 0.3 \leq 450^2$$

$$\frac{37.785}{d^4} + \frac{1.1025}{d^4} + \frac{3.87261}{d^4} \leq 450^2$$

$$\frac{42.76}{d^4} \leq 202500$$

$$d^4 \leq \frac{202500}{42.76}$$

from MDET

$$\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 \le S_{yt}^2$$

$$\frac{37.785}{d^2} + \frac{1.1025}{d^4} + \frac{6.4543}{d^4} \le 450^2$$

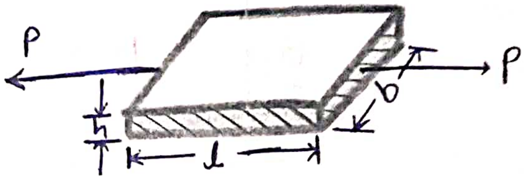
$$\frac{45.3418}{d^4} \le 450^2$$

$$d^4 \le \frac{202500}{45.3418}$$

Types of failure :

According to load,

failure due to tensile force



$$\sigma_T = \frac{P}{A} = \frac{P}{b \times h} \le S_{yt}$$

Qn. A mild steel rod is being pulled by the axial tensile force of magnitude 8kN. If the tensile strength of material is 2500 MPa. Find the diameter of the rod.

given, mat. - ductile (Mild steel)

$P = 8\ kN = 8 \times 10^3$, $S_{yt} = 250\ MPa$

$$\frac{P}{A} = S_{yt}$$

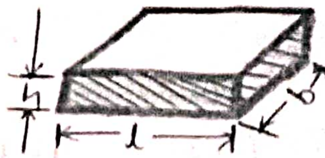
$$\frac{8 \times 10^3 \times 4}{\pi d^2} = 250$$

$$d^2 = \frac{8 \times 10^3 \times 4}{250 \times \pi} = 40.7436$$

$$d = 6.383\ mm$$

Failure due to shear force,

$$\tau = \frac{F}{b \times l} \le S_{sy} = \frac{S_{yt}}{2}$$



Qn. Ractangle block of dimension of 15×10×10 mm is Subjected to a shear force can with stand a shere force of 10kN on its square surface. Find the tensile strength of material.

$$\tau = \frac{F}{A} = \frac{10 \times 10^3}{10 \times 10} = 100$$

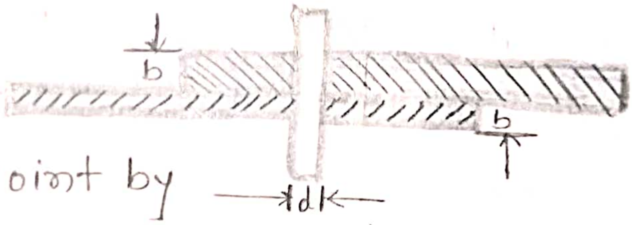
$$\frac{S_{yt}}{2} = 100$$

$$S_{yt} = 200 \text{ mPa}$$

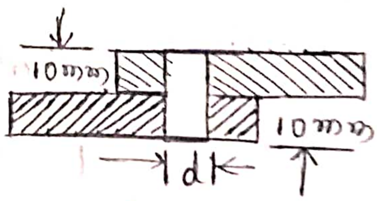
Failure due crushing load (Compressive load),

"crushing force is force between two objects in contact."

$$\sigma_c = \frac{P}{b \times d} \le S_{yt} = 2 S_{sy}$$



Qn. The metal plates are joint by nut and bolt as shown in figure. If the crushing strength of bolt is 0.5 GPa. Find diameter of the nut?



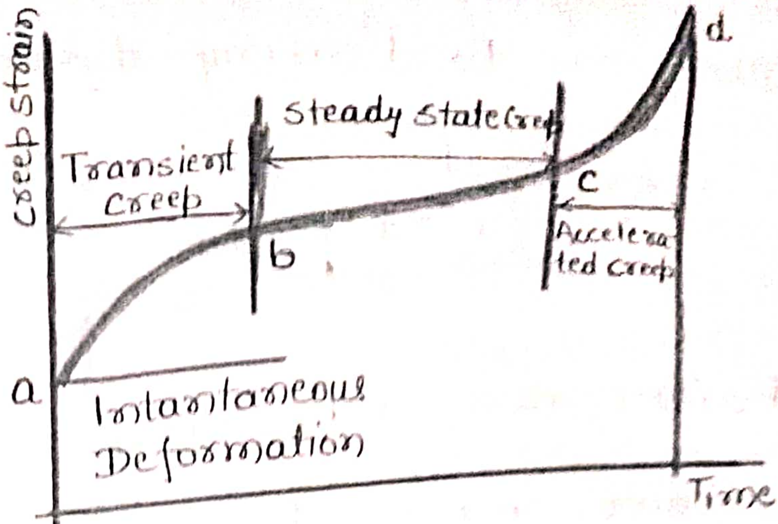
$$\sigma_c = \frac{P}{d \times b}$$

$$0.5 \times 10^3 = \frac{10 \times 10^3}{10 \times d}$$

$$d = \frac{1}{0.5} = 2 \text{ mm}$$

Regiodh (a-b)- Transient creep

"Strain rapidly increases but near point b strain rate decreases due to the strain hardening."



Creep curve

Region (b-c)- Steady state creep

"It happens due to the movement of dislocation."

Region (c-d) Accelerated creep

"Due to necking in material strain rate increases rapidly."

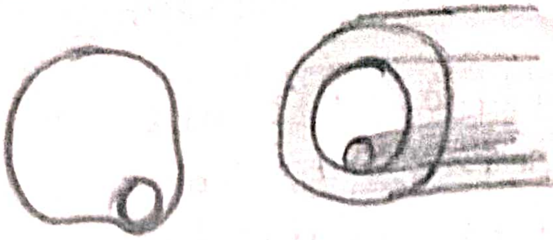
Creep :-

Creep is defined as slow and progressive deformation of the material with time under a constant stress(load). Creep deformation is a function of stress level and temperature, creep become important for component operating and elevated temperature. For example; steam or gas turbine blades, bolts and pipe in thermal power station.

Creep strength:- Creep strength of material is defined as the maximum strength/stress that the material can withstand for a specified period of time without excessive deformation.

Failure due to bearing dynamic load,

$$\boxed{\sigma_p = \frac{P}{l \times d}}$$



Failure due to bending,

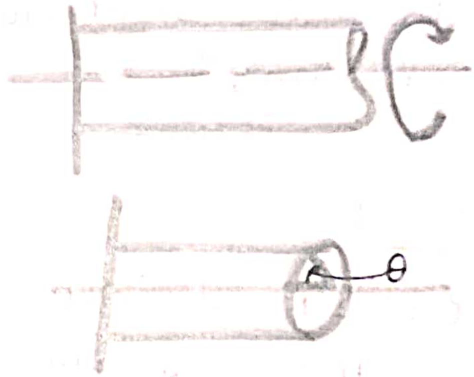
Bending equation, $\boxed{\frac{M}{I} = \frac{\sigma_b}{Y} = \frac{E}{R}}$

$$\boxed{\frac{I}{Y} = Z}$$

$\sigma_{b\,max} \leq S_{yt}$

$$\boxed{\sigma_b = \frac{MY}{I} = \frac{M}{Z} \leq S_{yt}}$$

Failure due to Torsion,



Torsion equation, $\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l}}$

$$\boxed{\tau_{max} = \frac{TR}{J} \leq S_{sy}}$$

$$\boxed{\theta_{max} = \frac{Tl}{GJ} \leq \theta_{permissible} \text{ (radian)}}$$

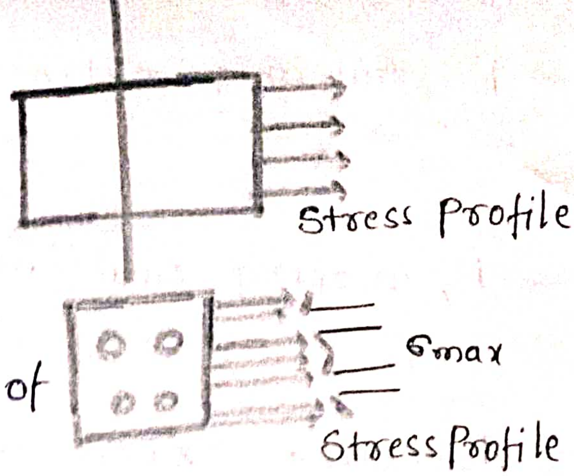
Creep Failure:

→ Permanent deformation over a period of time.
→ Stress under failure limit.

$\boxed{\text{Strain Rate} = \frac{\text{strain}}{\text{Time}}}$ If strain increases the strain rate decreases.
(Strain ↑, Strain rate ↓).

→ Due to strain hardening strain rate decreases.

Stress Concentration



Stress concentration is defined as the localization of high stress due to the irregularity present in the component and adrupt changes of cross-section.

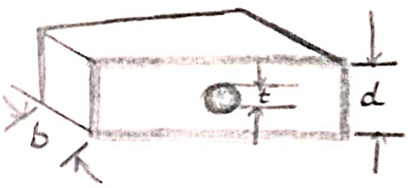
Stress concentration is determined by Photo Elasticity Method, in Photo elasticity method a specimen made up of epoxy resin is kept on the polariscope loaded at the edges.

Stress configuration factor,
It is used to accont the effect of variation in stress distribution.

$$k_{fc} = \frac{\text{max}^n \text{ stress in discontinuity}}{\text{nominal stress calcuted by elementry calculation}}$$

$$\boxed{k_{fc} = \frac{\sigma_{max}}{\sigma_o}}$$

$$\sigma_{max} = \frac{P}{(d-t)\cdot b}$$

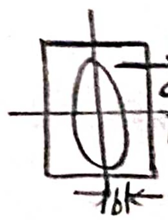


$$\frac{\sigma_{max}}{\sigma_o} > 1 \qquad , k_{fc} = 1$$

$$\underset{\text{(Design Stress)}}{S_{yt}'} = \frac{S_{yt}}{k_t}$$

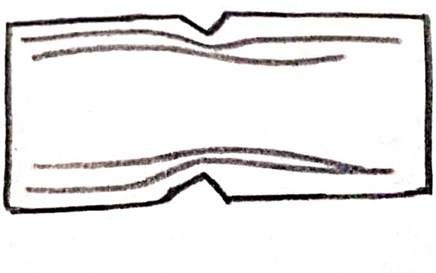
$$\boxed{k_{fc} = 1 + 2\left(\frac{a}{b}\right)}$$

where a - semi major axis
b - semi minor axis



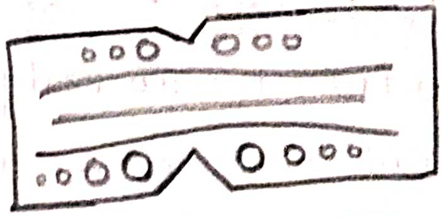
Remedy :

- Sudden dis continuity should be avoided.



→ By making extra notches.

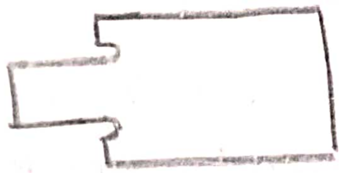
→ By making additional holes.

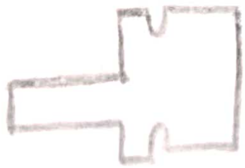


→ Removing excess materia,

→ Fillet radius,

→ Under cut





→ Notch

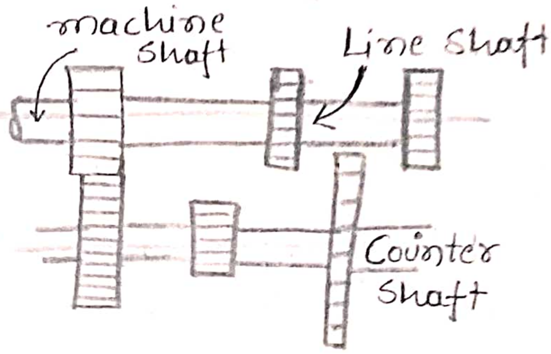
UNIT-3 Design of Shaft

"Shaft is a mechanical rotating member used for transmit the power."

Shaft

→ Machine shaft ; It is an integral part of machine example crank shaft.

→ Transmission Shaft ;
- Line shaft
- Counter shaft
- Over head shaft



Function ;—

1. Power Transmission
2. Support gear, pulley etc

Load on shaft :-

1. Torsion
2. Bending
3. Axial Force (Tensile or compressive)

Design of the shaft on the basis of,

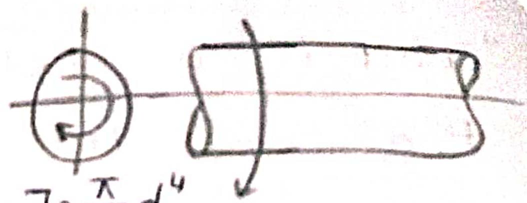
1. Strength - Induced strength/stress should be within permissible limit.
2. Rigidity - deformation should be within permissible limit.

Strength critaria;

1. Simple torsion
2. Simple bending
3. Combined torsion for bending
4. Combined torsion for bending with axial load.

1. Simple torsion

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l}$$



$J = \frac{\pi}{32} d^4$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\tau = \frac{TR}{J} = \frac{d/2 \cdot T}{\pi/32 \, d^4} = \frac{16T}{\pi d^3}$$

$$\boxed{\tau = \frac{16T}{\pi d^3} \le \tau_{permissible}}$$

$$P = \frac{2\pi N T}{60}$$

$$T = \frac{60P}{2\pi N}$$

$$d = \sqrt[3]{\frac{16T}{\pi \tau}}.$$

2. Simple Bending

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\sigma_b = \frac{My}{I} = \frac{M \times d/2}{\pi/64 \times d^4} = \frac{32M}{\pi d^3}$$

$$\sigma_b = \frac{32M}{\pi d^3} \le S_{yt} \quad \text{(Tensile or compressive)}$$

3. Combined Torsion and Bending

$\tau = \frac{16T}{\pi d^3}$ & $\sigma_b = \frac{32M}{\pi d^3}$ for solid shaft.

for solid shaft, $J = \frac{\pi}{32} d^4$

for hollow shaft, $J = J_1 - J_2 = \frac{\pi}{32}(D^4 - d^4)$

$$= \frac{\pi D^4}{32}\left(1 - \frac{d^4}{D^4}\right)$$

then, $\frac{T}{J} = \frac{\tau}{R}$

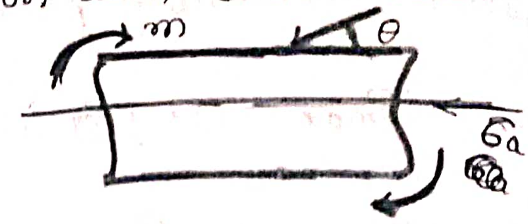
$$\tau = \frac{TR}{J} = \frac{T \cdot \cancel{D} \times \cancel{32}\,16}{\cancel{2}\,\pi D^{\cancel{4}}(1-k^4)} \qquad = \frac{\pi D^4}{32}(1-k^4) \qquad k = \frac{d}{D}$$

$$\boxed{\tau = \frac{16T}{\pi D^3(1-k^4)}}$$ for hollow shaft,

for hollow shaft compare to solid shaft,

$$\boxed{(J_1 - J_2) < J}$$

4. Combined bending and torsion with axial load.



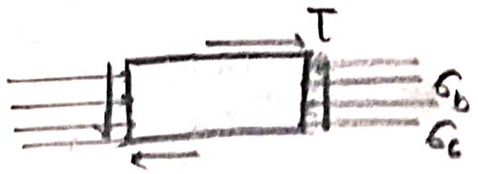
$\because \sigma = \sigma_b + \sigma_c$

$$= \frac{32M}{\pi d^3} + \frac{4P}{\pi d^2}$$

$$\boxed{\sigma = \frac{4}{\pi d^2}\left[\frac{8M}{d} + P\right]}$$

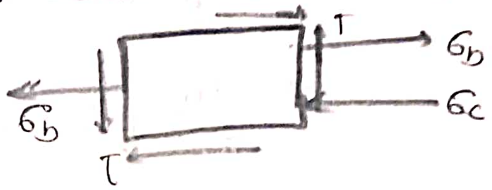
$$\boxed{\tau = \frac{16T}{\pi d^3}}$$

above neutral axis



$\sigma = \sigma_b + \sigma_c$ or σ_a

below neutral axis



$\sigma = \sigma_b - \sigma_c$ or σ_a

from maximum principle stress theory,

$$\sigma_1 \text{ or } \sigma_2 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= \frac{\cancel{4}2}{\cancel{2}\pi d^2}\left[\frac{8M}{d} + P\right] + \sqrt{\left(\frac{\cancel{4}2}{\cancel{2}\pi d^2}\left[\frac{8m}{d} + P\right]\right)^2 + \left(\frac{16T}{\pi d^2}\right)^2}$$

$$= \frac{2}{\pi d^2}\left[\frac{8M}{d} + P\right] + \sqrt{\frac{2}{\pi d^2}\left[\frac{8M}{d} + P\right]^2 + \left(\frac{16T}{\pi d^3}\right)^2}.$$

ASME Code for shaft design:

ASME stands for American society of mechanical engineeri

According to ASME,

for static load,

shaft without keyway,

$\tau_{per} = 0.3\, S_{yt}$ $\quad [S_{ut} = 0.5\, S_{yt}]$

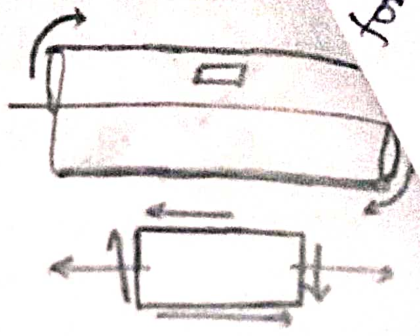
$\tau_{per} = 0.18\, S_{ut}$

shaft with keyways,

30% less than without keyways,

$= 70\%\ \tau_{per}$ (without keyways)

from maximum principle stress theory,



$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$= \frac{16M}{\pi d^3} + \sqrt{\left(\frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{16M}{\pi d^3} + \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

$$\Rightarrow \frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right]$$

$$\sigma_{max} \leq S_{yt}$$

$$\boxed{\frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right] \leq S_{yt}.}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$= \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

$$\tau_{max} < S_{sy}$$

$$\boxed{\frac{16}{\pi d^3}\sqrt{M^2 + T^2} < S_{sy}}$$

Equivalent moment (M_e): $\tau = 0$

$$\sigma_{max} = \sigma_b = \frac{32 M_e}{\pi d^3}$$

$$\frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right] = \frac{32 M_e}{\pi d^3}$$

$$\boxed{M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]}$$

Equivalent Torque (T_e):

$M = 0$

$$\tau_m = \frac{16 T_e}{\pi d^3}$$

$$\frac{16}{\pi d^3}\sqrt{M^2 + T^2} = \frac{16 T_e}{\pi d^3}$$

$$\boxed{T_e = \sqrt{M^2 + T^2}}$$

for dynamic load,

1. Strength ↓
2. Load ↑,

Equivalent Moment or Equivalent Torque;

k_bM - Combined fatigue and stress factor of bending.

k_bT - Combined fatigue and stress factor of torsion.

Equivalent Moment, $k_b, k_t \geq 1$.

$$\boxed{M_e = \frac{1}{2}\left[(k_bM) + \sqrt{(k_bM)^2 + (k_tT)^2}\right]}$$

Equivalent Torque,

$$\boxed{T_e = \sqrt{(k_bM)^2 + (k_tT)^2}}$$

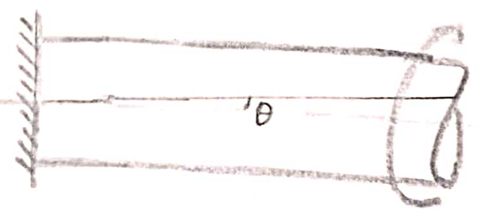
Rigidity Critaria For design;

$$\frac{T}{J} = \frac{G\theta}{l}$$

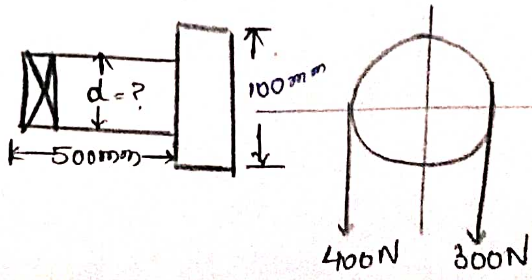
$$\boxed{\theta_{per} = \frac{Tl}{GJ} = \frac{5760\, Tl}{G\pi^2 d^2}}$$

Rigidity (Deformation of body)

$$\underbrace{\frac{\tau}{R} = \frac{T}{J}}_{\text{Strength (bearing of capacity).}} = \frac{G\theta}{l}$$



Qn. A pulley is overhanging from shaft as shown infigure. Tension at tight side and slack side are 400N and 300N respectively. The weight of pulley is 5 kg. Find the diameter of the shaft, given that tensile strength of shaft is 90MPa. and yield strength of the shaft is 50MPa. Combined fatigue and stress concentration factor of bending and torsion are 2.2 and 1.2 respectively. Consider factor of safety as 2.



given, $T_1 = 400N$, $T_2 = 300N$

$W = 5kg \times 9.81 = 49N$

$S_{ut} = 90 MPa$

$S_{yt} = 50 MPa$

$k_b = 2.2$, $k_t = 2.1$, $FOS = 2$

$T = (T_1 - T_2) r$

$= 100 \times 50$

$= 5000 Nmm$

$F = 400 + 300 + 49 = 749 N$

$M = F \times l = 749 \times 500$

$= 374500 Nmm$

According to ASME,

$0.18 S_{ut} = 0.18 \times 90 = 16.2 MPa$

$0.3 S_{yt} = 0.3 \times 50 = \underline{15 MPa}$

$\therefore \tau_{per} = 15 MPa$

$\tau = \frac{15}{2 (FOS)} = 7.5 MPa$

$\therefore M_e = k_b \times M = 2.2 \times 374500 = 823900 MPa$

$T_e = k_t \times T = 1.2 \times 5000 = 6000 MPa$

$T = \sqrt{Me^2 + Te^2}$

$= \sqrt{823900^2 + 6000^2}$

$= 823921.847 MPa$

$\tau = \frac{16T}{\pi d^3}$

$7.5 = \frac{16 \times 823921.847}{\pi d^3}$

$d^3 = \frac{13182749.55}{23.56} = 559539.4546$

$d = \sqrt[3]{559539.4546} = 82.4 \; \underline{\underline{mm}}$

Qn. A shaft is subjected to bending moment of 7.5 Nm & torsion of 10 Nm if permisible shear strength and yield strenth of shaft material is 40 MPa & 85 MPa respectively Find the diameter of shaft?

given,

$M = 7.5\ Nm$

$T = 10\ Nm$

$S_{yt} = 85\ MPa$

$S_{sy} = 40\ MPa$

$\tau_{per} = S_{sy} = 40\ MPa$, $\sigma_{b\,per} = S_{yt} = 85\ MPa$

$$T_e = \sqrt{M^2 + T^2}$$
$$= \sqrt{7.5^2 + 10^2}$$
$$= 12.5\ Nm = 12.5 \times 10^3\ Nmm$$

$$\tau = \frac{16\,T_e}{\pi d^3}$$
$$40 = \frac{16 \times 12.5 \times 10^3}{\pi d^3}$$
$$d^3 = \frac{200000}{125.663} = 1591.558$$
$$d = \sqrt[3]{1591.558} = 11.67 \cong 12\ mm$$

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$$
$$= \cancel{\frac{1}{2} \times 7.5 + 12.5 = 3.75 + 12.5}$$
$$= \cancel{16.25\ Nm}\ \times\ \times$$

$$= \frac{1}{2} \times (7.5 + 12.5)$$
$$= \frac{20}{2} = 10 \times 10^3\ Nmm$$

$$\sigma_b = \frac{32\,M_e}{\pi d^3}$$
$$85 = \frac{32 \times 10 \times 10^3}{\pi d^3}$$
$$d^3 = \frac{320000}{267.035} = 1198.3447$$
$$d = \sqrt[3]{1198.3447} = 10.621 \cong 11\ mm$$

The diameter of shaft should be 11 mm.

Key :

key is mild steel component used to restrict the relative motion between shaft and hub.

The strength of key is always less than strength of shaft.

Types of key;

1. Sunk key : That key which is fully inserted in or between shaft and hub. known as sunk key.

a. Ractangular Sunk Key,

It is ractangular tapered shaped key.

It is easy to remove or insert between keys and shaft.

b. Square key,

It is modified form of ract-angular key.

" $t = w = \frac{d}{4}$ ".

c. Parallel key,

It is also ractangular key who is not tapered in dimension.

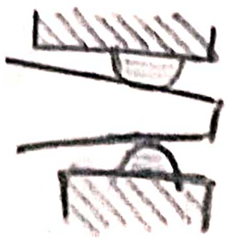
d. Gib-head key,

It is easy to remove or insert.

e. Feather key,

It is either fixed in shaft or hub.

It is used to transmit torque only.



f. Wood-druff key,

It allows angular misalignment or adjust itself.

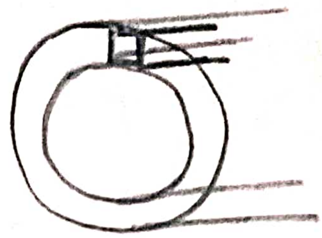
It can used in tapered shaft or hub.

2. Saddle key :-

In saddle key power transmission can done due to friction only. Its design can done on 'μ'.

$T = F \times r$

$=$



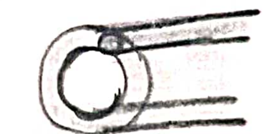
(i) Full saddle

(ii) Hollow Saddle.

3. Tangent key :-

In this type of key power transmission in one direction like paddle in cycle.

4. Round key :-



It is rounded in shape and its cross-section is round or circular.

5. Spline key :-

"It provide more power transmission in less area."

Design of Sunk key ;

Power transmission can done from shaft to key and key to hube.

$$P = \frac{2\pi NT}{60},$$

$$\tau_s = \frac{16T}{\pi d^3}, \quad T = \frac{\pi \tau_s d^3}{16}.$$

(i) failure due to shear force,

$$\tau_K = \frac{F_K}{Wl}$$

$$F_K = \frac{T}{r} = \frac{T}{d/2} = \frac{2T}{d}$$

$$\tau_K = \frac{2T}{Wld}$$

$$T = \frac{1}{2}\tau_K Wld.$$

Step (ii) Failure due to crushing

$$\sigma_c = \frac{F_K}{\frac{t}{2} l} = \frac{2T/d}{l t/2}$$

$$\sigma_c = \frac{4T}{d t l}$$

$$T = \frac{1}{4} \sigma_c d.t.l.$$

procedure,

Step-1. To find the torque of Shaft

Step-2. Find the diameter of shaft.

Step-3. Find the length, width, and thickness of key.

Step-4. Check the failure due to crushing and shear.

Qn. Find the diameter of shaft transmitting 20 kW power at 300 rpm, and design the parallel sunk key if shear strength and crushing strength of key is 30 MPa or 50 MPa respectively also if yield or ultimate tensile stregth of shaft is 250 MPa or 380 MPa respectively.

Couplings

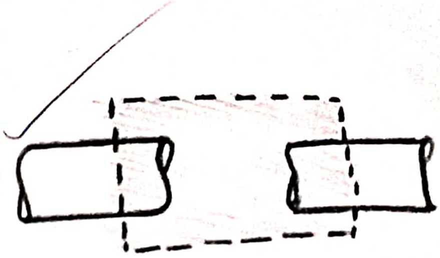
04.05.24

Coupling means to couple different shaft

By couple,

(i) Increase portability

(ii) Machine become less heavy



(iii) Cost effective

(iv) Easy to maintain.

Types of couple;

1. Rigid coupling : It is used when shaft are perfectly align.

Type;

a). Muff / Sleeve coupling

b). Split muff coupling or

Compressive maff coupling

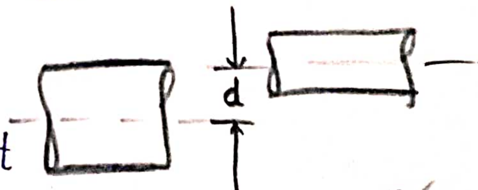
c). Flange coupling

(i) Protected type

(ii) Unprotected type.

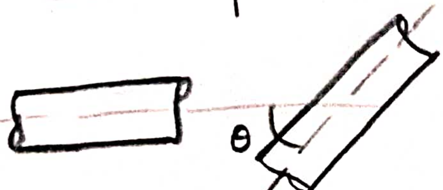
(iii) Marine type

2. Flexible coupling : Where is having some mis-alignment between two shaft, the mis-alignment may be angular alignment as well as axial alignment.



Types; (a) Angular alignment

(b) Axial alignment.



(i) Bush Pin Coupling

(ii) Universal Coupling / Hook's Joint (Angular)

(iii) Oldhen coupling (Axial).